Linear folded V-shaped stripes

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Abstract

This paper presents research to find a computational method for creating freeform structures consisting of simple linear folded V - shaped stripes.
A geometric algorithm produces a series of stripes that form regular and irregular reticular structures on a given surface (Fig. 1). This algorithm enables the approximation of single to double curved surfaces. The V- section form of the stripe has advantages over other known folded stripe systems by adding rigidity to the stripes and whole structure. Indeed, simple linear folded stripes can be considered as half reverse folds. Being rectangular in unrolled condition, the stripes undergo no torsion when folded.
This system can be classified as a post defined open stripe system (Maleczek, Geneveaux 2011).

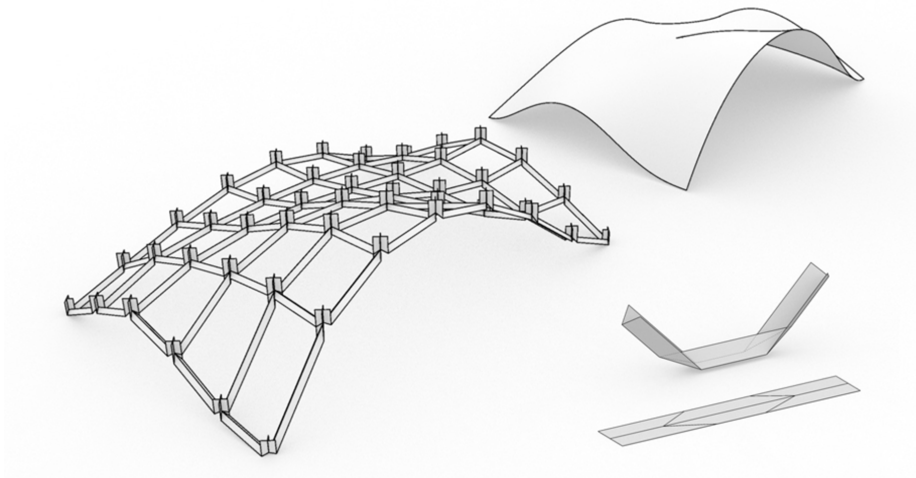

Figure 1: a surface structuralised with L-shaped stripes, and an example of its genotype

V-SHAPED STRIPES

The genotype of this technique is a popular and often used element in these types of structures. It has been described as reverse folded stripes (Buri 2010), as rigid isometric origami (Klett; Drechsler 2011), or as unit with isotropic vertices (Tachi 2009). In its unrolled position it forms a rectangular stripe, with three folds in the length along the middle axis, and two additional folds on each side (Fig. 2).

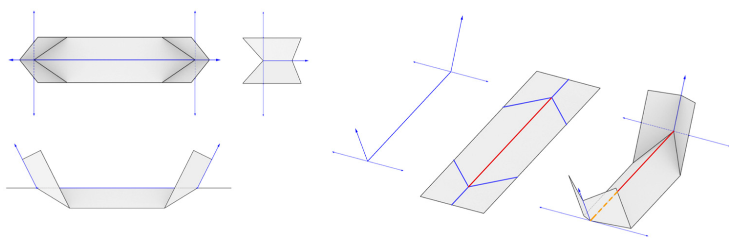

Figure 2: a regular V-shaped stripe

The centred fold along the middle axis is a mountain fold, while all other folds are valley folds. The angle between the side folds and the middle fold has to have a variation of 90 degrees; otherwise the stripe can not be folded. More than two side folds can be formed on the same stripe. This folded stripe has one degree of freedom that enables the variation of dihedral angles between the stripe's planes. In a stripe, all dihedral angles formed by the planes separated by the middle fold are equal, only their direction is inverted.
This folding technique is usually used in a surface approach, by multiplying the number of mountain and valley folds on a unique corrugated surface. The authors are interested here in the assembly of V-shaped stripes into reticular structures, using star-like nodes. This strategy can be seen as an alternative to an approach where the structure consists of large folded panels, instead providing a reticular structure that consists of relatively small folded elements.

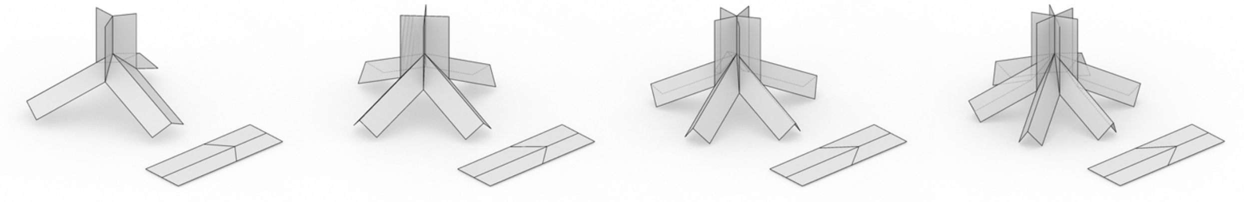

Figure 3: regular star-nodes with 3,4,5 and 6 elements

The number of stripes that can be assembled in a star-like node is variable (Fig3). To have a V-shaped section of the base module, a minimum of three elements must be assembled. In this paper; regular nodes, semi-regular nodes and irregular nodes will be distinguished. A regular node is defined by equal angles between the middle folds, thus all dihedral angles are also equal. In regular nodes the number of assembled elements directly defines the dihedral angle. Nodes with various angles lead to different dihedral angles, and can be described as irregular nodes.
In this paper different techniques and algorithms for surface approximation will be presented. Curved surfaces will be approximated by polygonal faces. The advantages and limits of this approach will be presented from a geometrical as well as from a structural point of view.

Stripe-Elements and their Generation

V-shaped stripes can be defined as a variation of simple linear folded stripes. A simple folded stripe can be considered as a half of a V-shaped stripe, cut along the middle fold. The main difference is the assembly technique. The faces of each stripe can be defined as connection- and contact-segments. While contact segments are connected together to manage the assembly of stripes, connection segments connect two contact segments, belonging to the same stripe. In this particular system, the connection-segments are joined to each other through a fold along the middle axis of the stripe. To assemble stripes together in a reticular structure, a reverse fold is created at each end of the stripe. In this configuration, each stripe will have a minimum of one middle fold in its length, forming the "Middle-Axis"(Fig. 4) and four side folds. In most cases, each contact segment is connected to a contact segment, which belongs to another stripe. Therefore the number of contact segments of each stripe defines the number of stripes it is connected to. In this paper the majority of the described stripes are connected to four other stripes.

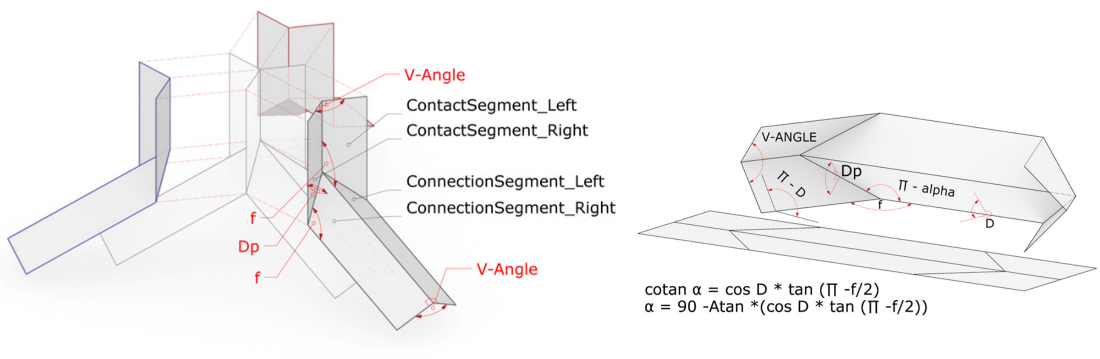

Figure 4: a regular node assembly with the stripe elements and their correlation

As V-shaped stripes are defined by the assembly method, it is beneficial to generate the stripes from points and vectors (Fig. 5). Each stripe can be generated from two points and three vectors connected to each point. For each stripe, the middle axis can be generated between these two points. In each case, two vectors are representing the middle-axis of the neighboring segments, and one vector, the common direction of all folds connecting segments on the star-like node. This direction is identified as the Pin-Direction.

The V-angle is the bisector of the angle between the middle-axis of the neighbouring segments. If all neighbouring middle axis have the same angle relative to its neighbours measured in the pin direction, the star-like node can be described as a regular node. For the regular node the correlation between the different angles can be described as in Fig. 4. If all nodes in a reticular structure are the same regular nodes, this structure can be described as a regular tessellation.
If the stripes are generated from a mesh, then a mesh edge with its neighbouring mesh edges and a direction at its vertices will be required to generate a stripe along each edge.

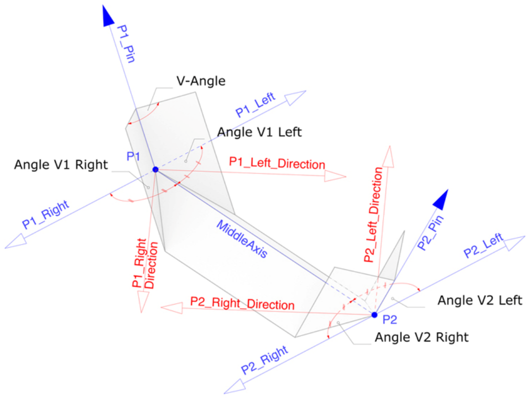

Figure 5: Elements needed for the stripe creation

Regular Star-like Nodes and their corresponding Tessellation

As all dihedral angles in a regular star-like node have to be equal, a grid of regular nodes must fulfil this relation in all nodes. There are three classical regular tessellations, which will be described here (Fig. 6). The dihedral V-Angles can be defined by a complete circle (360°) divided by the number of middle-axis meeting at each node. This establishes triangulated grids with an equal dihedral angle of 60 degrees (360°/6) , rectangular grids with an equal dihedral angle of 90 degrees (360°/4) , and hexagonal grids with an equal dihedral angle of 120 degrees (360°/3). With this formula, the V-angle for each stripe connected in a regular node can be calculated. These angles must be have an equal measurement from the pin of each node.

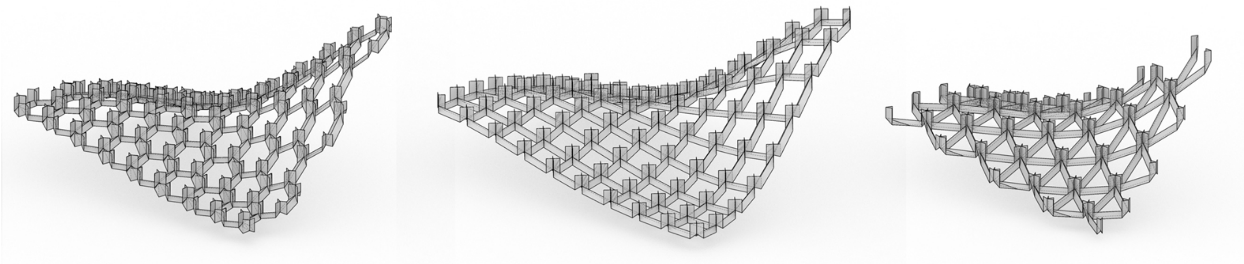

Figure 6: from left to right: a hexagonal, a quadrahedral and a triangulated regular tessellation on the same surface

Therefore it is useful to create regular tessellations with pin directions that intersect either in a point or in infinity. In the special condition, of meeting in infinity, all pins are parallel. The advantage of regular grids with regular star-like nodes lies in the fact that the V-Angles of all stripes are equal.

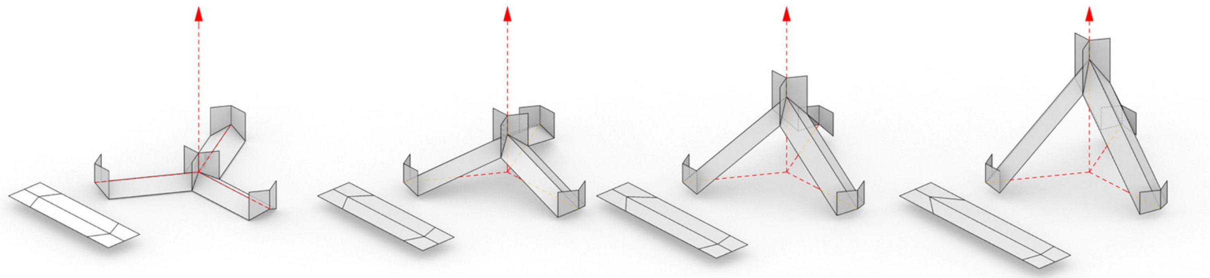

Figure 7: a regular node with changing height, by a constant V-Angle

Regular tessellations allow for a very interesting technique to approximate doubly-curved surfaces. As all V-angles within a regular tessellation are depending on the number of elements in the node, and must be measured in a plane, where the pin direction is equal to the surface normal, the point in the node itself can be moved along the pin direction, with no influence on the V-Angle, but a change in all angles alpha of the folds measured to the middle axis and the folding angle Dp in these folds (Fig. 7), depending on the change of the angle f.

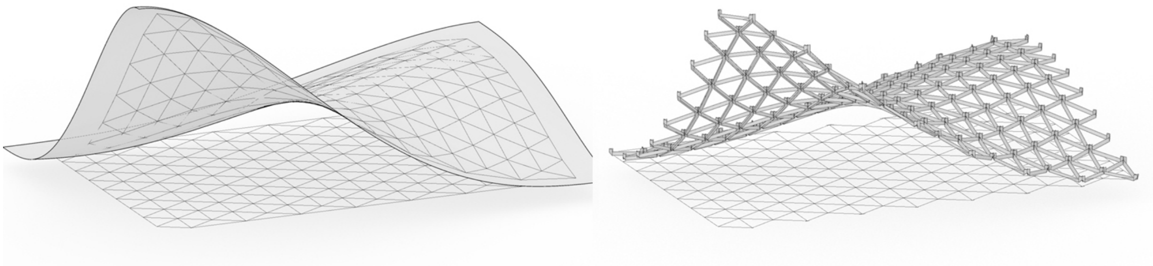

Figure 8: a triangulated regular grid projected to a surface.

One simple strategy to approximate surfaces with regular grids can be a projection of a regular grid of lines to a surface in pin direction (Fig. 8). Projected to a sphere this strategy will lead to an icosahedron, if the grid is triangulated. In this icosahedron not only the V-Angle is equal in all stripes, but also the fold angles Dp are equal. A flat surface structuralised with a regular grid, will generate equal stripes, that have the same V-Angle and equal fold-angles Dp in every fold. As soon as the approximation of a double curved surface is needed, the V-Angle remains the same in all stripes, but the folding angle Dp and the correspondent angle α will be different in most folds of the stripes.

One possible strategy to minimize the number of different folding angles could be placing limitation on the height distance of neighbouring points, to fixed values, so there will only be a limited number of folding angles. With the adjustment of the steps only, without taking into account its neighbours, the number of different angles is reduced enormously. Here the density and the allowed steps will define the number of different angles within the stripe system (Fig 9). One issue here is the decision on the "smoothness" of the resulting structure.

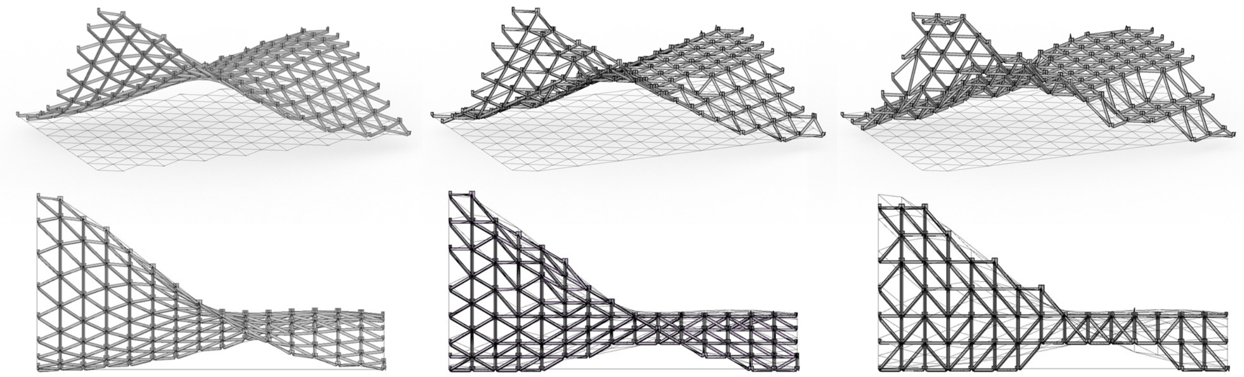

Figure 9: adjustment of the steps in the direction to approximate a surface with a fixed number of different stripes

The method described above, has some limitations concerning the feasibility and approximation of possible surfaces. One limitation is for parallel-stripe systems is the direction of the surface normals in relation to the pin direction (Maelczek 2010). If the angle between the surface normal and the pin direction is larger than 90 degrees, the regular star-like node will no longer work within the system. In other words, surfaces with undercuts will not work with regular tessellations (Fig. 10).

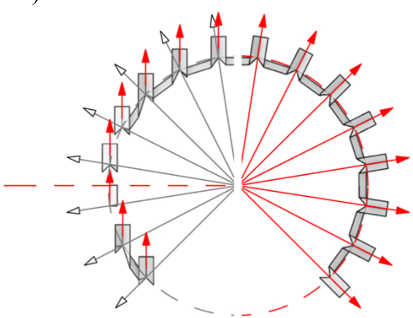

Figure 10: the limit of parallel pin directions (left) and the possible solution with radial pin directions (right)

One possible solution would be the development of a stitching method. In order to create buildable structures, it seems to be beneficial to avoid surface normals directions that get close to this orthogonal condition. Some surfaces allow an approximation with pin directions aligned to a centreline (Fig. 11). Therefore a radial grid around this centreline is projected to the surface. Here the undercut in radial pin direction poses the same problem as in parallel condition.

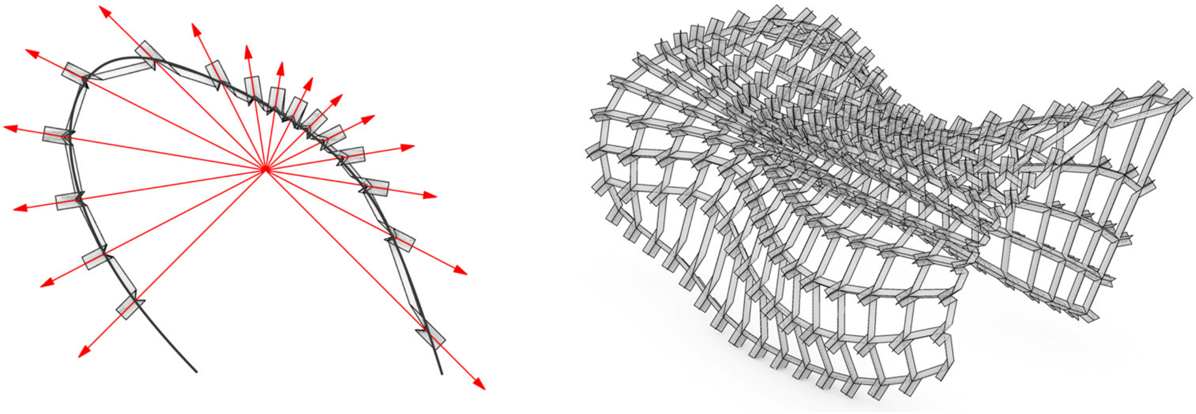

Fig 11: an example for the limit of parallel pin directions and a surface structuralised with radial orientation of the pin directions

Another problem lies in the surrounding border of regular grids. Here in the case of the nodes, either the number of folds will differ, to the regular nodes in the grids, or the number of members is the same, but the angles will differ from the regular star-node.

Genotype-variations and their corresponding tessellations

The Genotype described above works on a wide variety of surfaces but is, as described, limited and restricted for parallel or centred pin-directions in combination with regular equal angled middle-axis. In order to extend the degrees of freedom of this reticular folding system, there are several possibilities and approaches. As one main concern of the paper is to keep the stripe rectangular in its unrolled condition, the authors propose two variations of the genotype. While the first solution works only under very strict boundary conditions, the second one enables a wide variety of forms and possible tessellations by introducing up to two "double-folds"(Maleczek, Genevaux, Ladinig 2012).

Semi-regular star-like-nodes

In this approach, the number of folds within the stripe member will not be changed. Therefore it is necessary that the angle and vector relationship follows very strict rules. The dihedral angles V in both pin directions have to be parallel, and the diagonal facing adjacent directions P1_Left and P2_Right as well as P2_Left and P1_Right have to be parallel. If these three conditions are fulfilled, it is possible to keep the number of folds constant. Therefore, one fold between two contact segments must be rotated. If this solution is chosen, the node can be described as irregular, caused by the fact, that the connected contact faces, will not be connected with the full surface area. In comparison to a stripe based on a regular node, this approach will also rotate the V-Angle around the baseline, if the stripe is creating one regular node, and one semi regular node (Fig 12).

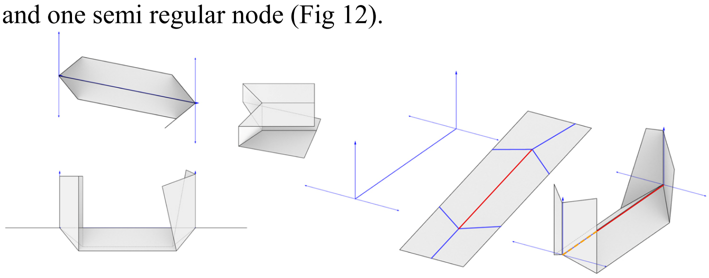

Figure 12: a semi-regular V-shaped stripe

One big advantage of this system is the assembly of semi-regular tessellations (Fig. 13). A semi regular tessellation is a grid, where all nodes are connecting the same number of stripes, and the V-Angle for all members in the structure is the same. The main difference is that the adjacent middle axis of the neighbouring stripes, are no longer defining the V-Angle. In order to keep the V-Angle constant, it must be calculated from fixed directions in all nodes within the tessellation. If the middle axis of a stripe is in-between these fixed directions, this semi-regular V-shaped stripe can be used. Therefore semi-regular grids can be seen as deformed regular tessellations.

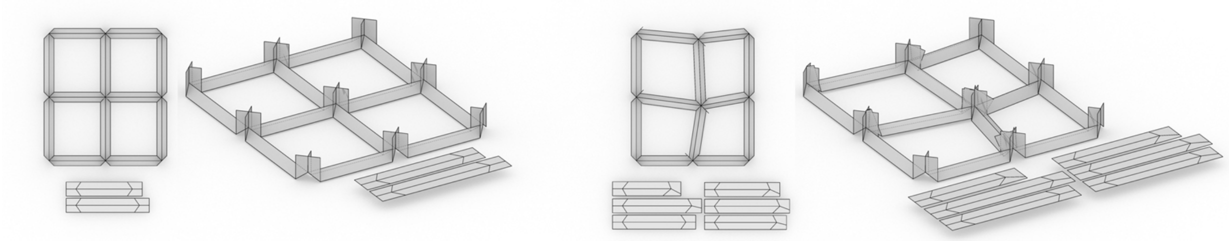

Figure 13: a regular tessellation (left) and a semi regular tessellation (right)

Here the star-like nodes exhibit the same angles as in a regular grid, but the middle-axis must no longer be in the exact centre of the V-Angle. All stripes have one degree of freedom, and are rectangular in unrolled position. In order to approximate surfaces with stripes that have only one degree of freedom, semi-regular grids offer a good solution. As semi-regular tessellations, can be seen as deformed regular tessellations, the reticular structure, can consist of both, regular V-shaped stripes and non-semi-regular V-shape stripes (Fig 14).

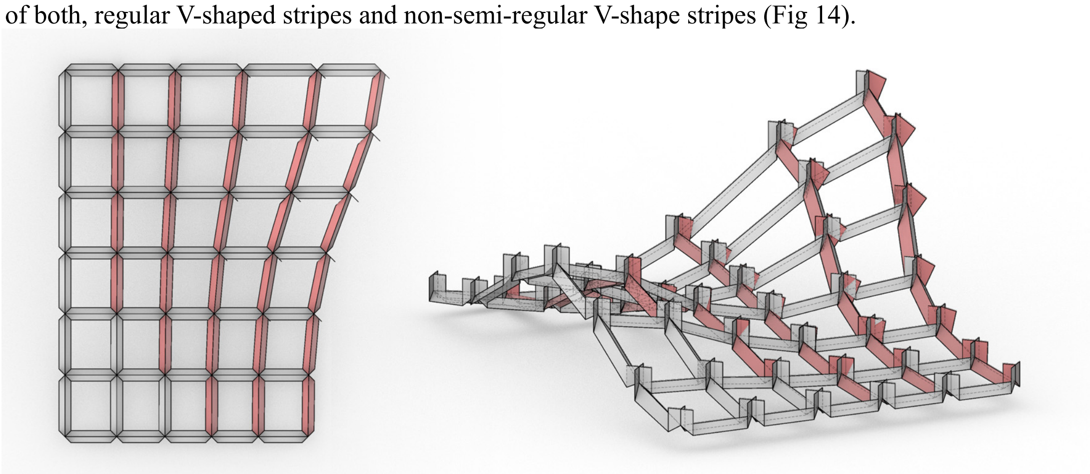

Figure 14: a deformed regular tessellation with the semi regular stripes in red

Irregular Star-like Nodes and their corresponding tessellations

Regular stripe configurations have, per definition, equal pin directions and equal angle relations. If these relationships are no longer fulfilled, the stripes need, in most cases, additional folds. Depending on the relationships of the different angles, one or more double folds are necessary in order to create a linear folded stripe. These double folds, already described for mesh-based stripes (Maleczek,Genevaux,Ladinig 2012), enlarge the possible node configurations, and grids of reticular structures enormously. A double fold in V-shaped stripe allows, for the connection of nodes with differing numbers of members and therefore different angles. As soon as regular or semi regular grids change their pin directions from regular to irregular, this genotype will be very useful (Fig 15).

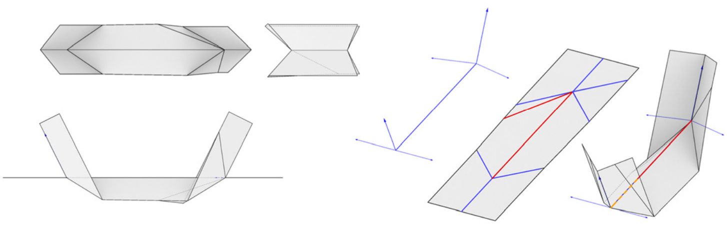

Figure 15: an irregular V-shaped stripe

For architectural use, this method can not only be used to extend tessellation strategies on a given surface, but can also be used to approximate mesh geometries. For a structuralisation of a mesh geometry, the mesh edges will represent the middle axis of the stripe members, and the vertices will represent the nodes. Here, not only triangulated meshes, but also quadrilateral meshes can be used to generate a reticular structure. The pin directions within the structure itself can either be defined through the vertex normal of the mesh or through other constraints. As there is a body of research in the optimisation of freeforms for architectural use, meshes provide a powerful tool to generate stripe based structures from (Sheppard 2011). From form-finding to planar mesh faces a wide variety of solutions are available, for the generation of meshes. These techniques and tools are not only provided as theoretical knowledge but also as tools and add-ons for existing software packages. In other words, the use of mesh geometry for the production of V-shaped stripes opens up a wide field of approaches to generate geometry and form.

The Structural implications of Liner folded V-shaped stripes:

There is much to support the use of the V-shaped stripes. Whilst the benefits of a geometry which is easily constructed out of flat sheets with few liner folds is obvious, the structural consequences and benefits are not so trivial to grasp. In this section some structural implications of the reticulation strategy will be outlined.
Firstly the generation of the structural members in the form of V-shaped structural sections inherently provides for moment resisting depth. While it is generally desirable for such reticulated forms to work as shells with forces travelling in the plane of the surface of the structure, invariably out of plane forces are present which incur bending in the structure that it then must resist. The structural depth afforded by the V fold effectively provides this resistance. There are some impacts on this benefit however brought about by pin direction. The optimal case for structural depth is where the pin directions are parallel with the local surface normals, here we develop the largest out of plane depth, conversely pin angles which approach tangent to the surface have close to zero geometric depth measured normal to the surface and represent low performing structural designs (Fig 10). Thus this gives further incentive to choose pin directions effectively.
Additionally the V shape section is a relatively stable geometry during axial compression, it should also be noted however that if the grid is sufficiently large bucking might become issue. In this case there would be considerable benefit in closing off the V section to provide a more stable section.

The connection detail of the stripe nodes have the advantageous quality of the top folds of the stripes joining to a single point with respect to the stripe centrelines. This allows for simple resolution of forces promoting better shell action of the structure. Looking in more detail the stripes directly bare on to each other allowing for easy fixing strategies. It should be noted that in the case there are multiple edges, which have significantly different angles from the node pin direction to each other then there will be significantly varying depths of the beams. This results in varying second moments of area for stripes made of same width material, and if substantial this could lead to unequal out of plane stiffness and less even and effective distribution of forces.
One concern which would require further investigation is the potential for the structure to generate stress concentrations around joints in the folds at the nodal points, however such investigations are out of the scope of this paper, but could potentially be mitigated by appropriate reinforcement which should be considered for latter study.
Due to the relatively high geometrical freedom of the system to place nodal positions, there is much scope to apply relaxation techniques (Williams 2001) to optimally place the nodal positions and minimise the issues associated with reticulated grids in general and problems specific to the V-Stripes detailed above.

Combined assembly strategies

The presented V-shaped stripe generations lead to a powerful tool when the different systems are combined, or used with additional constraints. Reticular structures that consist of combined regular and irregular grids, in order to achieve a minimisation of different folding angles for the entire structure, can be imagined. Meshes can be utilized that were not only generated through form-finding techniques but also took into consideration a reduction in the production time, by reducing the number of folds needed.

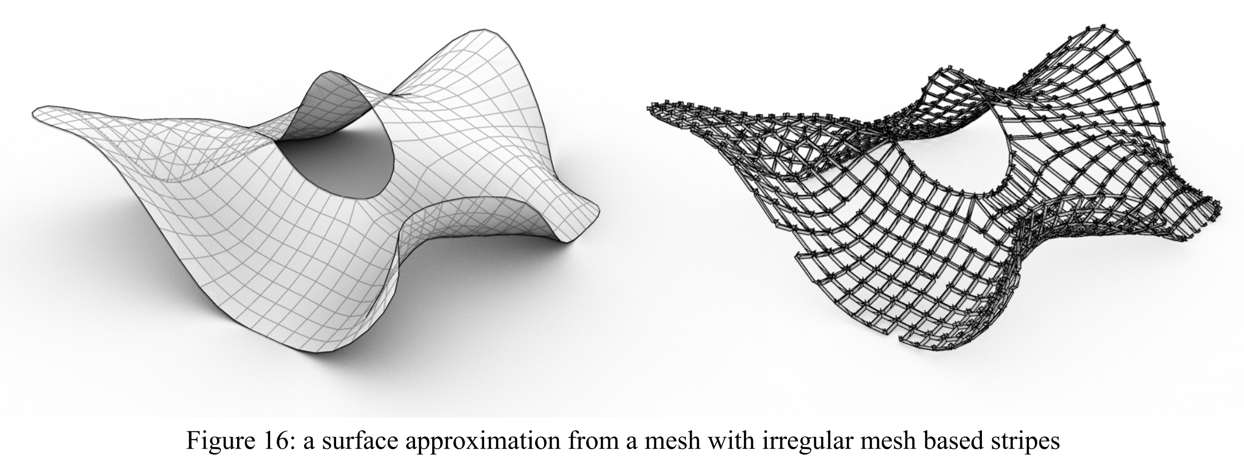

Figure 16: a surface approximation from a mesh with irregular mesh based stripes

Besides the combination of different systems approximating one given Surface, V-shaped stripes can also be used to approximate double layered surfaces with closed stripe systems (Maleczek; Geneveaux; 2011). This strategy could be used for the creation of doubly curved sandwich panels at a small scale, and for the creation of double layered building structures for freeform spaces.
An ongoing area of research is the investigation into the structural abilities of the different systems. This exploration, in combination with the production issue, will extend the possibilities of the presented structures, and seem to be very promising for the future.

Another recently started research on this system is the use of curved folded stripes, to minimize the number of folds, and extend the structural abilities of this system (Fig. 17).
Recent findings such as this show that the vast range of potential of developing and using such systems is just being uncovered and will only continue with further research.

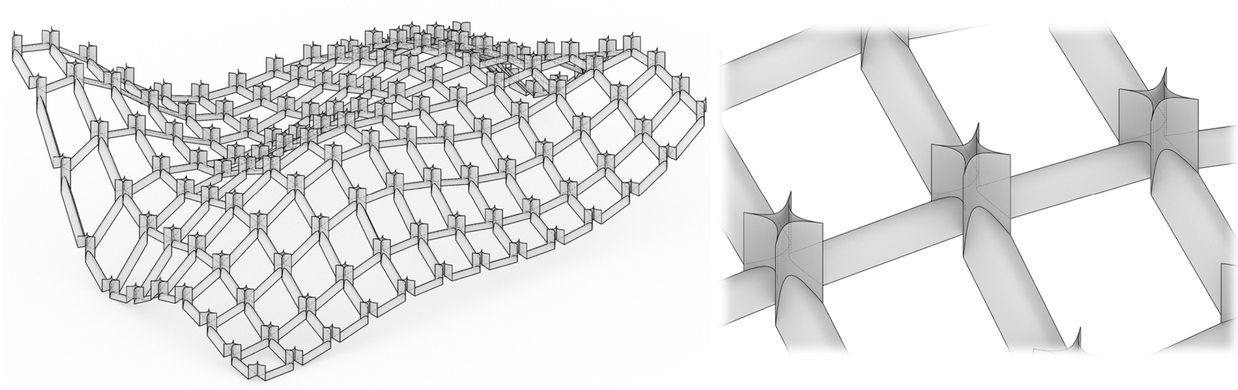

Figure 17: a surface approximation with curved folded stripes

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